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TBMs AND THE FLAMING DATUM PROBLEM

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TBMs AND THE FLAMING DATUM PROBLEM

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ABSTRACT

Theater Ballistic Missile launching systems are vulnerable just after a missile is launched because the missile's track can be extrapolated backwards to the location of the launcher. The situation is similar to one where a submarine torpedoes a ship, thus creating a "flaming datum" near which ASW forces may concentrate a search for the submarine. This report describes how some simple analytic methods adapted from ASW can be applied to the task of locating the TBM launcher.

1. THE TBM PROBLEM

SCUD launches during Desert Storm were a significant nuisance to the Allies, and could have been worse had the missiles been armed with other than conventional warheads. SCUD missiles are launched from Transporter Erector Launcher vehicles (TELs), the kind of system that can be expected with any Theater Ballistic Missile (TBM). TELs normally attempt to avoid detection, since they are subject to air attack if caught in the open. Launching the missile requires the TEL to be in the open, so the Allies spent considerable time and effort in "SCUD hunting" – attempting to find and destroy the TELs. This effort was not notably successful, (Cohen, 1993) so in the aftermath there has been some effort to explain why and perhaps fix whatever the problem was, since similar situations involving TBMs could occur in the future. This report is devoted to a simple model of one method of dealing with TBMs, the "flaming datum" attack that may succeed in destroying the TEL right after the launch of a missile.

One would prefer, of course, to find the TEL *before* the missile is launched, rather than after. If the TELs have sufficient area to roam in, however, or if they are difficult to distinguish from innocent traffic, it may not be possible to search the available area rapidly enough to have an appreciable effect. There is an AntiSubmarineWarfare (ASW) precedent for this situation. Submarines are difficult to detect without some kind of initial cue as to location, so one strategy for protecting ships from torpedo attacks is to wait until an attack occurs, which establishes a "flaming datum" near which there must be a submarine, and then react quickly to detect and attack the submarine. In doing so one is protecting all the ships that the submarine would have attacked in the future, if not the current victim. Such tactics are effective against submarines because each submarine would attack many ships if left alone; sinking a submarine right after a torpedo attack is almost as good as sinking it beforehand, statistically speaking. These tactics are also potentially effective against TELs, since a TBM launch is easily detected and extrapolated back to where the TEL must have been when it occurred. The only problem is to get to the launch site quickly and effectively. The chances of doing this are the subject of sections 2 and 3 of this report.

Nothing has been said above about the possibility that TELs may have shelters of some kind that protect them either from observation or attack. Such shelters could do much to tilt the game in favor of the TEL force. In the worst case the shelters would protect the TELs from air attack, and TBMs could be launched from the shelter or so near it as to require very little time in the open. In that case very little can be done from the air. In intermediate cases shelters might themselves be subject to air attack, or might merely protect TELs from observation, or significant travel away from the shelter might be required to launch a TBM. Analysis is possible in these intermediate cases, but it should be emphasized that there are no shelters or protective emplacements of any kind in the model described below, where the TEL's only hope after launch of a TBM is to become lost in an area so large that further search for it is pointless. If the environment is such that a TEL, after launching a missile, says to himself, "Now I have to get away from here in a hurry before searchers arrive", then the methods described below are applicable. If the TEL instead says, "Now I have to hurry back to the shelter", then they are not applicable.

2. FLAMING DATUM ATTACKS

Assume that a launch takes place at time 0, and that the launching vehicle (hereafter the "target") immediately proceeds away from the launch site with speed U. The target necessarily remains inside a circle with radius Ut at time t after the launch, the Farthest-On-Circle or FOC. At time t_1, a searcher arrives and begins searching in the FOC at speed V_S and with sweep width W. The sweep width W is assumed to hold everywhere in the region, so there is no specific place for the target to hide. The target may nonetheless escape detection because the area of the FOC expands quadratically with time whereas the area covered by the searcher expands only linearly. A probabilistic model of detection can be based on this observation. In this model detections are assumed to happen in a nonhomogeneous Poisson Process where the rate of detection $\lambda(t)$ is the ratio of the rate of covering area ($V_S W$) to the area of the FOC at time t:

$$\lambda(t)=V_S W/(\pi U^2 t^2). \tag{1}$$

If the searcher searches between t_1 and t_2, the average number of detections $n(t_1, t_2)$ is the integral of $\lambda(t)$ between those limits:

$$n(t_1, t_2)=V_S W/(\pi U^2)\,(1/t_1-1/t_2). \tag{2}$$

Since the actual number of detections is a Poisson random variable, the probability of at least one detection is $1-\exp(-n(t_1, t_2))$, the desired detection probability. Limited testing of this model in an abstract situation where military officers played the roles of target and searcher in 295 replications is in agreement with it. Figure 1 is taken from Washburn (1989).

Formula (2) is much more sensitive to t_1 than to t_2. Assume for simplicity that t_2 is so large compared to t_1 that it can safely be assumed to be infinite (note that $n(t_1, t_2)$ does not approach infinity with t_2 – even searching "forever" will not necessarily detect the target). In that case the detection probability is

$$\mathrm{PD}(t_1)=1-\exp(-V_S W/(\pi U^2 t_1)). \tag{3}$$

Formula (3) will be assumed to govern detections in the next section.

3. AVERAGING THE DETECTION PROBABILITY

Formula (3) applies only when the time late t_1 is given. In practice the time late will vary depending on how near the nearest searcher is to the launch (only the nearest searcher is assumed to respond even if several happen to detect or are informed of the launch). We will use the upper case letter T to refer to time late in this section to emphasize the fact that it is random (but some nonrandom quantities such as W are also capitalized).

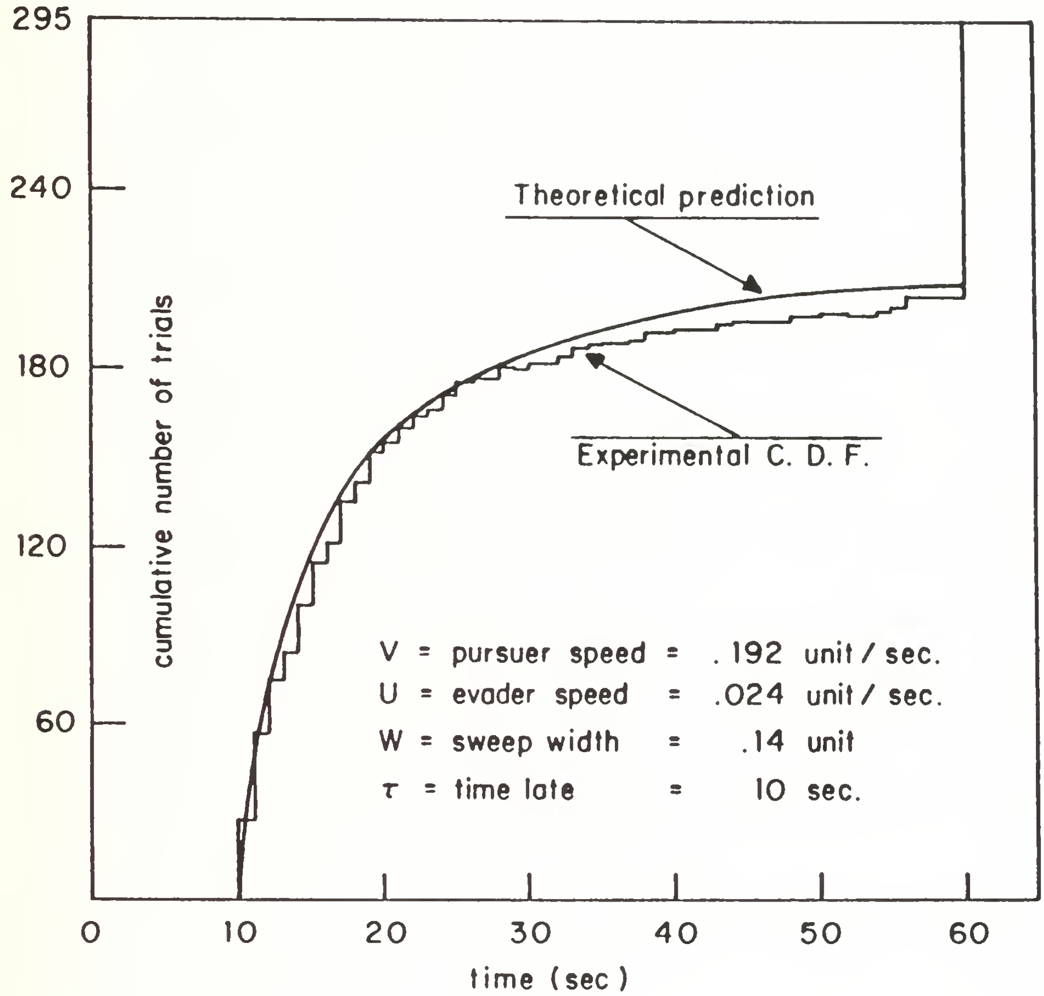

Figure 1

Let d be the average density of searchers per unit area. Typically d would be calculated by some formula like

$$d = Nf/A \tag{4}$$

where N is the number of searchers assigned to patrol the region in which targets can be expected, A is the area of that region, and f is the fraction of the time during which patrolling actually occurs. If a given searcher can patrol for 8 hours out of every 24 and if launches can happen anytime, then f is 1/3. If launches happen entirely at night (12 hours out of 24), then f would be 2/3 because the searchers would patrol only at night. If launches could happen anytime, but if the sweep width W were different between day and night, then determination of f would require solution of a two-person-zero-sum game.

A rough analysis can be made by assuming that d is the density of a two-dimensional Poisson field of searchers whenever the launch occurs, and that launches are so widely separated in time as to be considered independent events. Probably the latter assumption is worse than the former, since TELs are in reality motivated to make multiple simultaneous launches that will exhaust the patrolling searchers (as well as any defense that might be mounted against the missiles). In any case, the Poisson assumption determines the distribution of S, the distance from the launch to the nearest patroller. The event $(S>s)$ is the event that the number of patrollers in a circle of radius s about the launch is 0. Since the average number of patrollers in that circle is $\pi d s^2$, $\text{Prob}(S>s)=\exp(-\pi d s^2)$. Since T is related to S by $T=S/V_T$, where V_T is the transit speed of the closest searcher, the density function of T can be determined by differentiation of the cumulative distribution function to be

$$f_T(t)=2\pi t d V_T^2 \exp(-\pi d t^2 V_T^2). \tag{5}$$

T turns out to be a Rayleigh random variable. The average detection probability in a flaming datum attack is then

$$E(PD(T)) = \int_0^\infty \left[1-\exp\left(-V_S W/\left(\pi U^2 t\right)\right)\right] f_T(t)\,dt. \tag{6}$$

By letting

$$x = V_S V_T W \sqrt{d/\pi}\,/U^2, \tag{7}$$

and substituting $u=\pi d t^2 V_T^2$ in (6), equation (6) can be expressed in dimensionless form as

$$E(PD(T)) = 1-\int_0^\infty \exp\left(-\left(u+x/\sqrt{u}\right)\right)du. \tag{8}$$

Let (8) be called simply $P(x)$, the detection probability as a function of x. Figure 2 shows the function $P(x)$, calculated numerically, using MATLAB (1990).

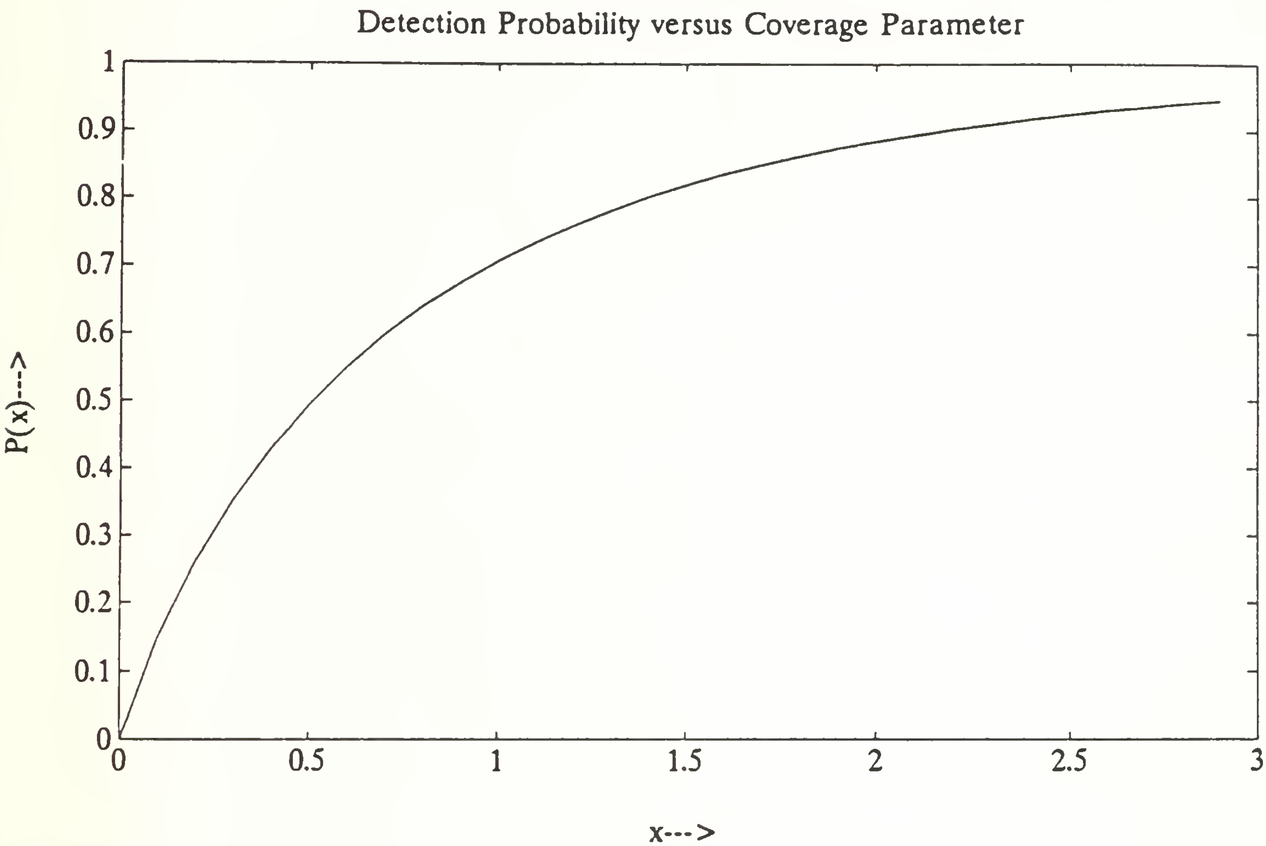

Figure 2

If the TBM launch is detected by some system other than the responding aircraft, then some allowance must be made for a communications delay c. The average response time without the delay is $E(T)=1/(2V_T\sqrt{d})$. Define an "equivalent transit speed" V_T' to be such that $1/(2V_T'\sqrt{d}) \equiv c + 1/(2V_T\sqrt{d})$, so that

$$V_T' = V_T/(1+2cV_T\sqrt{d}). \qquad (9)$$

Then replacing V_T by V_T' in calculating x makes a rough correction for the communications delay. A more accurate analysis would acknowledge that the total response time including the communications delay is no longer a Rayleigh random variable when c is included.

In summary, although there are a wide variety of input parameters that influence the detection probability, it is only the dimensionless coverage parameter x in (7) that is ultimately of any importance. Given x, the detection probability is a simple graphical lookup. Conner, et al. (1993) give a more general model of TEL detection that would accept this probability as an input.

4. AN EXAMPLE INVOLVING SCUD DETECTION

Except for W, which depends on the sensors involved and has therefore been chosen arbitrarily, the following assumptions are roughly characteristic of detection of SCUD launches within a country the size of Iraq by a fleet of 4 aircraft:

$$
\begin{aligned}
N &= 4 \text{ aircraft} \\
f &= .1 = \text{fraction of the time actually on patrol} \\
A &= 100{,}000 \text{ square miles} \\
V_S &= 100 \text{ mph} \\
V_T &= 300 \text{ mph} \\
W &= 10 \text{ miles} \\
U &= 20 \text{ mph}
\end{aligned}
$$

These parameters result in a coverage parameter x=.846, from which one would conclude that 1–P(.846)=.65, so 65% of the launches should result in ultimate detection by a pursuing aircraft.

If there were a communications delay c=15 minutes=.25 hour, then the equivalent transit speed V_T' according to (9) would be 231 mph. This would reduce x to .651, and 1–P(.651)=.58. Without the communications delay the average response time is already 50 minutes, so adding 15 minutes to it does not make a lot of difference.

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